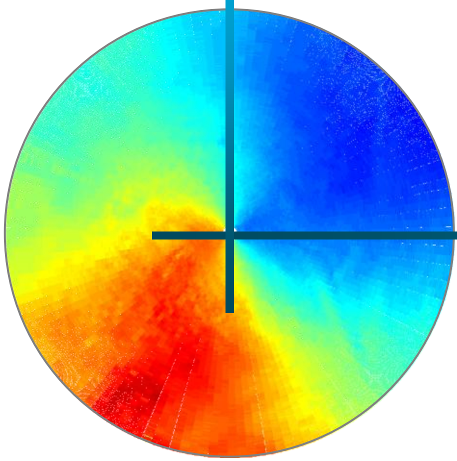


FM-CW Atmospheric radar

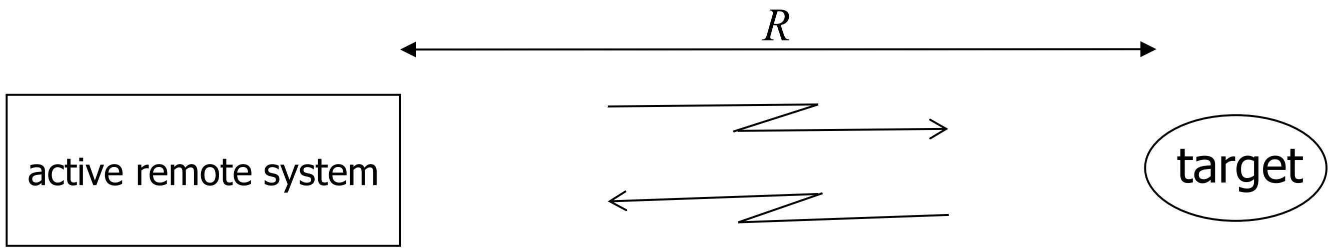
Herman Russchenberg, Tobias Otto, Christine Unal

- The Active Remote Sensing Equation
- Principles and Applications of FMCW Radars
- Processing

The Active Remote Sensing Equation

- is an analytical expression for the received power in an active remote sensing equation, i.e. radar or lidar
- merges all the knowledge about the system (relevant system parameters), the propagation path, and the objects that are remotely sensed
- is frequently applied for active remote sensing system:
  - design
  - conversion of the received power into a meaningful measurement, i.e. an observable that solely depends on what is remotely sensed
  - calibration
  - performance analysis

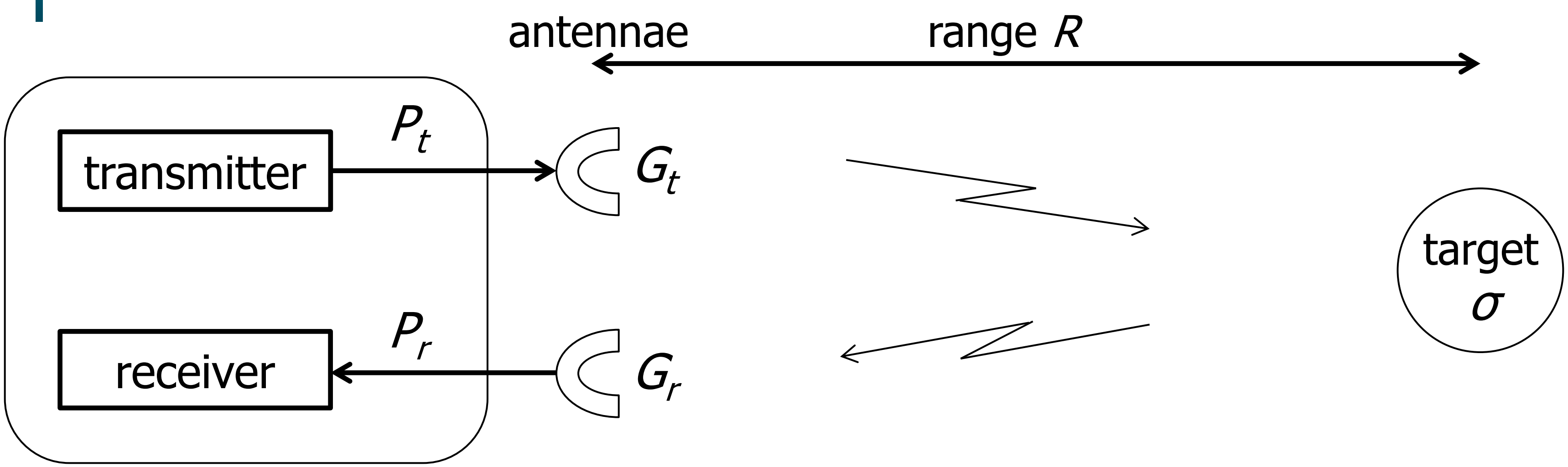
The Active Remote Sensing Equation



$$P_r = C \cdot G(R) \cdot B\left(R\right) \cdot T\left(R\right)$$

- C: active remote sensing system constant
- $G(R)$: range dependent measurement geometry
- $B(R)$: target characteristics (backward-scattering)
- $T(R)$: transmission term (attenuation)

Radar Equation for a Point Target

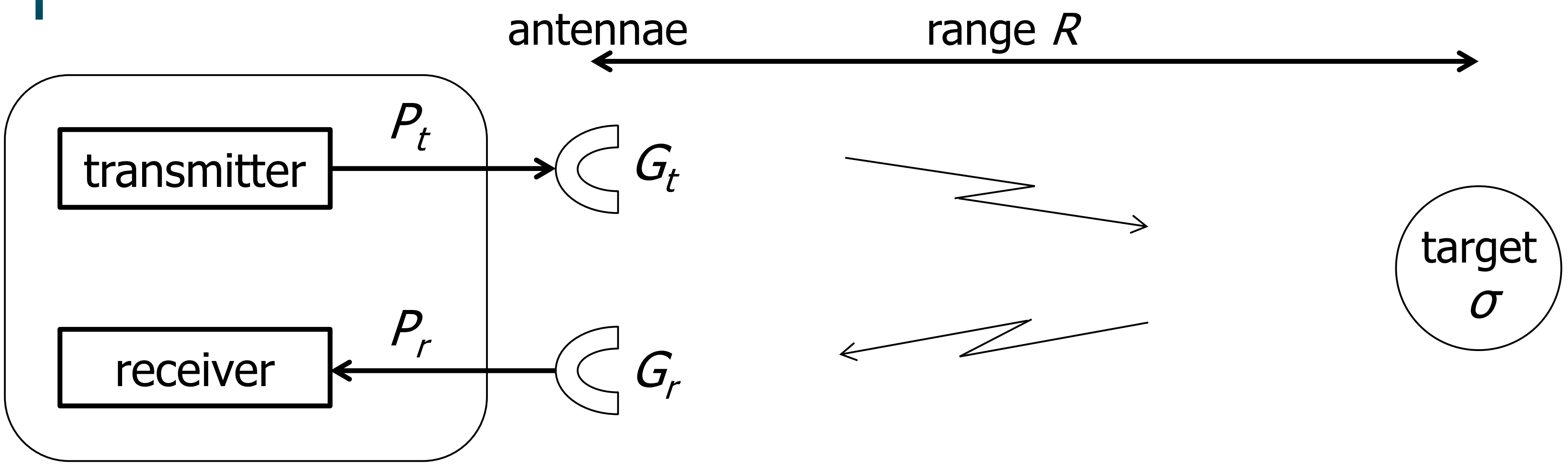


P_t .. transmitted power (W)
G_t .. antenna gain on transmit
R .. range (m)
σ .. radar cross section (m^2)
G_r .. antenna gain on receive
P_r .. received power (W)

$$P_r = \frac{P_t}{4\pi R^2} \cdot G_t \cdot \sigma \cdot \frac{1}{4\pi R^2} \cdot \frac{G_r \lambda^2}{4\pi}$$

TU Delft

Radar Equation for a Point Target



P_t .. transmitted power (W)
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σ .. radar cross section (m^2)
G_r .. antenna gain on receive
P_r .. received power (W)

$$P_r = \frac{P_t G_t G_r \lambda^2 \cdot \sigma}{(4\pi)^3 \cdot R^4}$$

From Point to Volume Targets

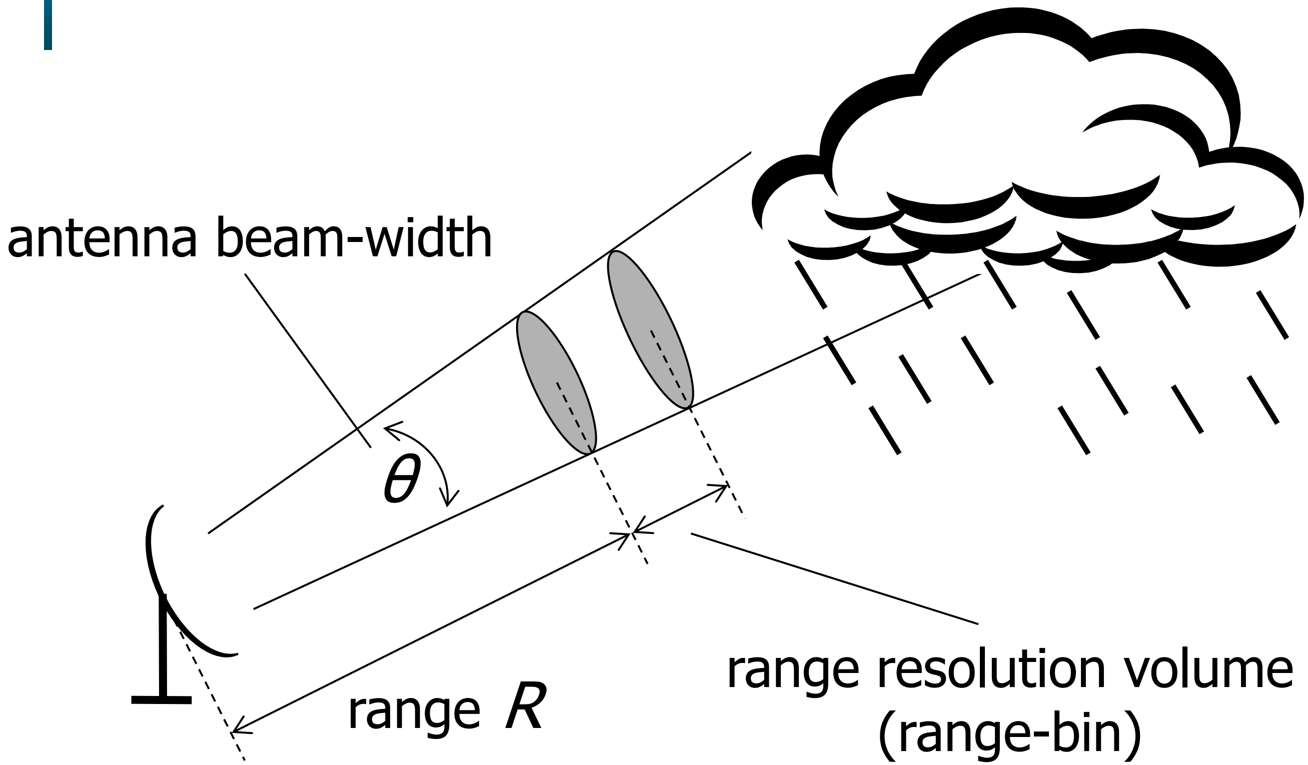
- the radar equation for a point target can be customised and expanded to fit the needs of each application (e.g. moving target inidication, synthetic aperture radar, etc.)
- radar has a limited spatial resolution, therefore weather radars can not observe single hydrometeors instead they *always* measure a volume filled with hydrometeors

  → volume target instead of a point target
- replacing the radar cross section with the sum of the radar cross sections of all scatterers in the volume (range-bin):

$$\sigma = \sum_{\text{range-bin}} \sigma_i$$

TU Delft

Radar Resolution

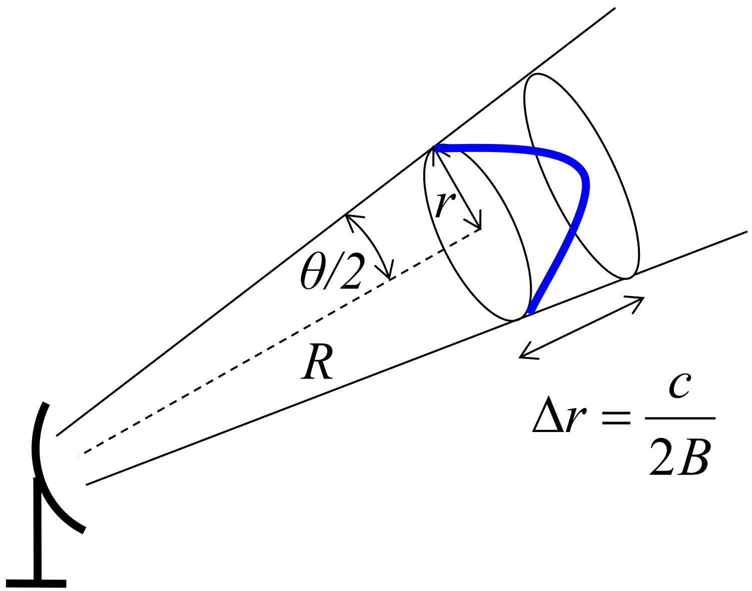


$$\Delta r = \frac{c}{2B}$$

c .. speed of light
B .. bandwidth of the transmitted signal

→ Δr is typically between 3m – 30m,
and the antenna beam-width is between 0.5° - 2° for atmosphere radars

Radar Equation for Volume Targets



$$V = \pi \cdot r^2$$

$$V = \pi \cdot \left[R \tan\left(\frac{\theta}{2}\right) \right]^2 \cdot \frac{c}{2B}$$

tan(α) ≈ α (rad) for small α

$$V = \pi \cdot R^2 \frac{\theta(\mathrm{rad})^2}{4} \cdot \frac{c}{2B} \cdot \frac{1}{2 \ln 2}$$

volume reduction factor due to Gaussian antenna beam pattern

Radar Equation for Volume Targets

$$\bar{P}_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \cdot V \sum_{\text{unit volume}} \sigma_i$$

$$\bar{P}_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 R^4} \cdot \frac{\pi \; R^2 \theta^2 c}{16 \ln 2 B} \cdot \sum_{\text{unit volume}} \sigma_i$$

Radar Cross Section σ

- ➔ Im case hydrometeors are small compared to the wavelengths used in weather radar observations: wavelength 3 mm ⇔ max. 40 mu cloud droplet diameter

- ➔ Rayleigh scattering approximation can be applied; radar cross section for dielectric spheres:

$$\sigma_i = \frac{\pi^5 |K|^2 D_i^6}{\lambda^4}$$

D .. hydrometeor diameter
λ .. radar wavelength
$|K|^2$ .. dielectric factor depending on the material of the scatterer

Radar Equation for Weather Radar

$$\bar{P}_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 R^4} \cdot \frac{\pi R^2 \theta^2 c}{16 \ln 2 B} \cdot \sum_{\text{unit volume}} \sigma_i$$

$$\bar{P}_r = \frac{\pi^3 P_t G_t G_r \theta^2 c}{1024 \ln 2 B \lambda^2} \cdot |K|^2 \sum_{\text{unit volume}} D_i^6 \cdot \frac{1}{R^2}$$

radar constant

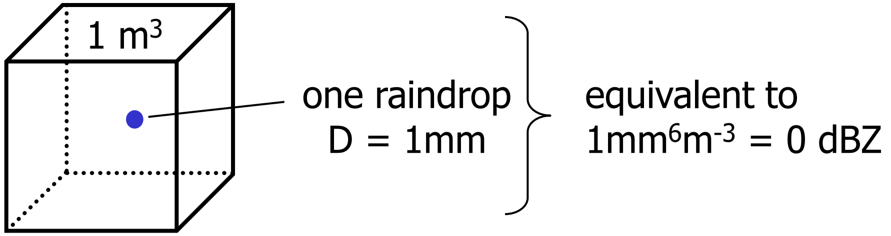
radar reflectivity factor *z*, purely a property of the observed precipitation

Radar Reflectivity Factor z

$$z = \sum_{\text{unit volume}} D_i^6 \left(\frac{\text{mm}^6}{\text{m}^3} \right)$$

→ spans over a large range; to compress it into a smaller range of numbers, engineers prefer a logarithmic scale

$$Z = 10 \log_{10} \left(\frac{z}{1\text{mm}^6\text{m}^{-3}} \right) (\text{dBZ})$$



raindrop diameter	#/m³	Z	water volume per cubic meter
1 mm	4096		2144.6 mm³
4 mm	1		33.5 mm³

Knowing the reflectivity alone does not help too much.
It is also important to know the drop size distribution.

Principles and Applications of FMCW Radars

Tobias Otto

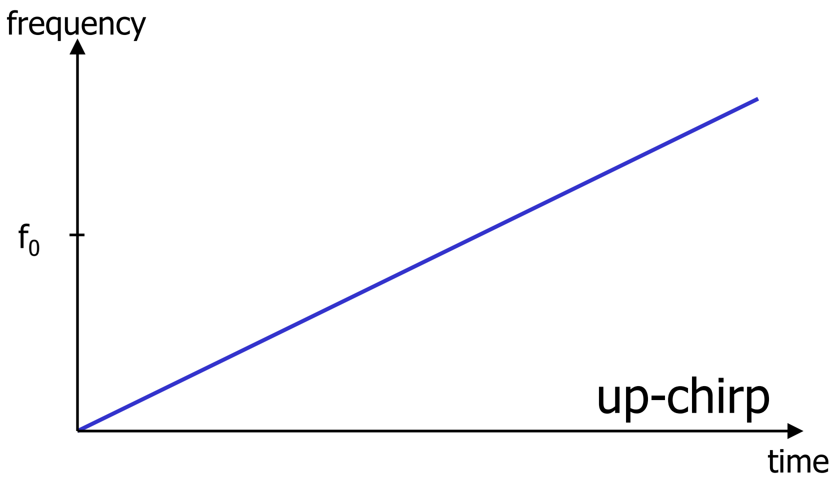
Principle of FMCW radar

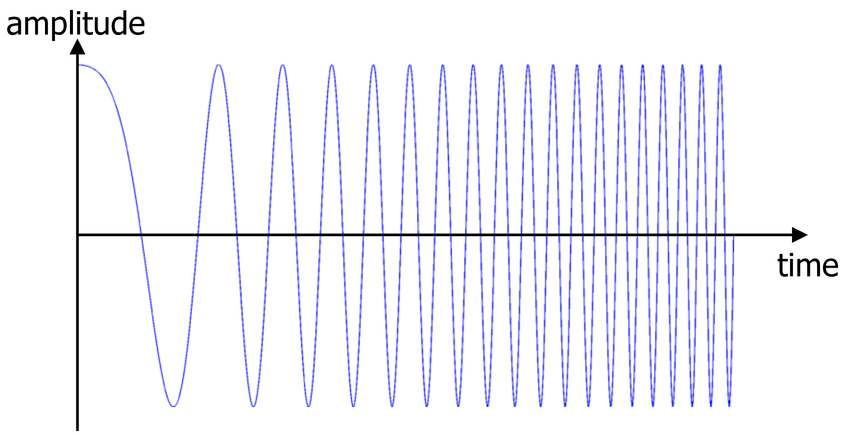
frequency-modulated continuous-wave

A radar transmitting a continuous carrier modulated by a periodic function such as a sinusoid or sawtooth wave to provide range data (IEEE Std. 686-2008).

Modulation is the keyword, since this adds the ranging capability to FMCW radars with respect to unmodulated CW radars.

We will concentrate in this talk on linear FMCW radar (LFMCW).





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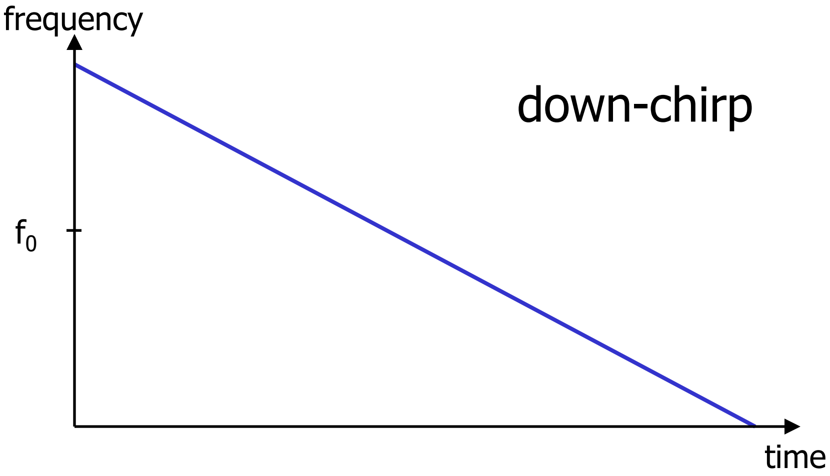

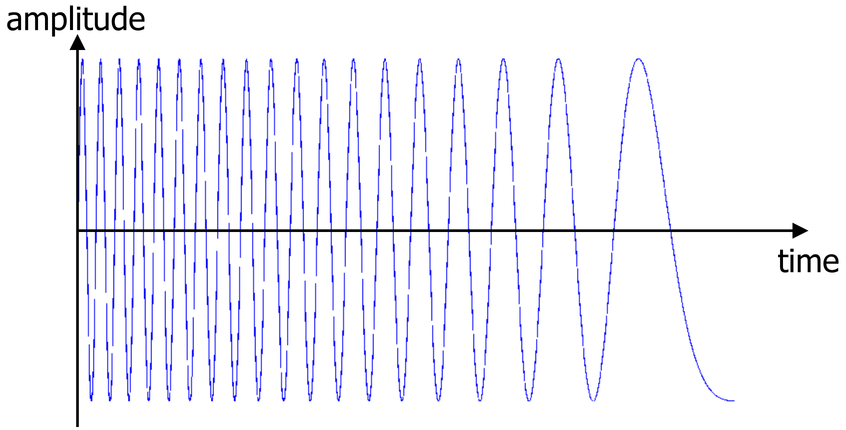

Principle of FMCW radar

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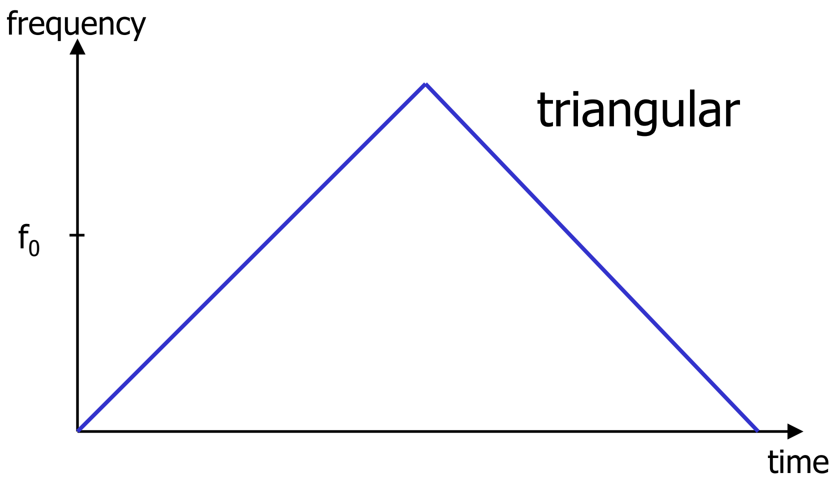
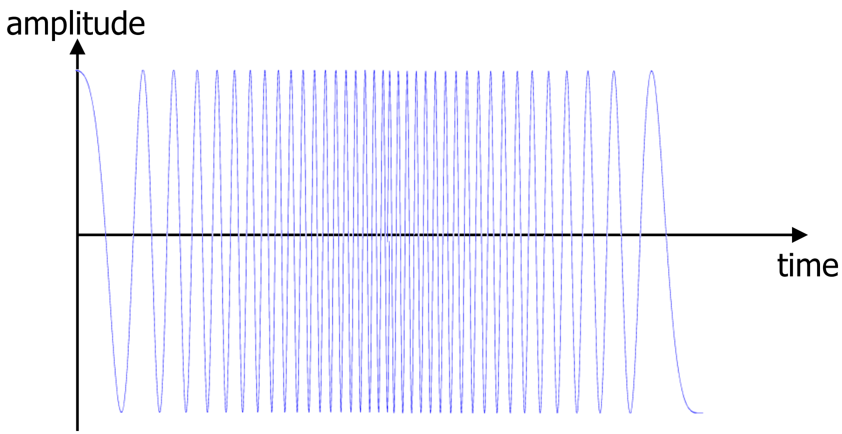
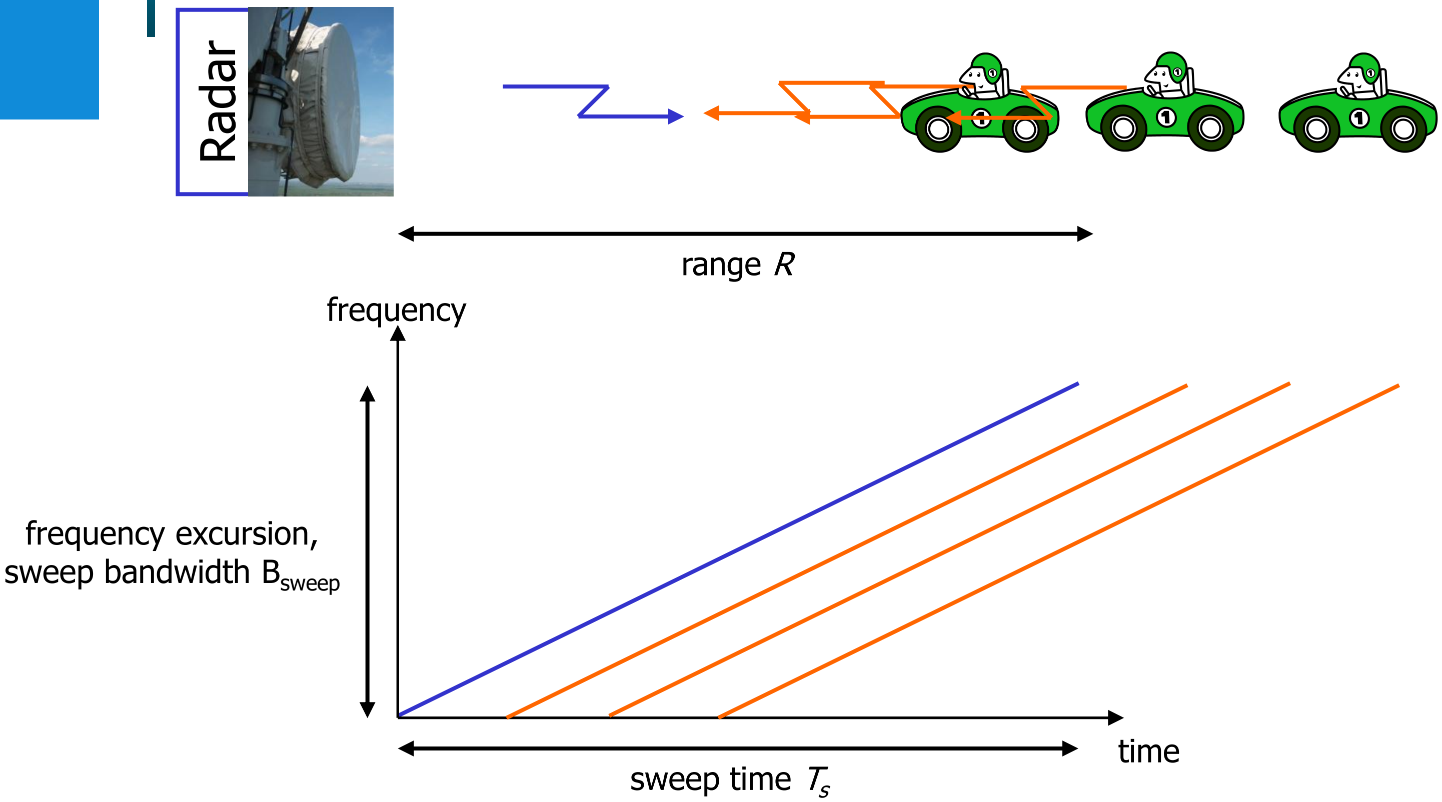
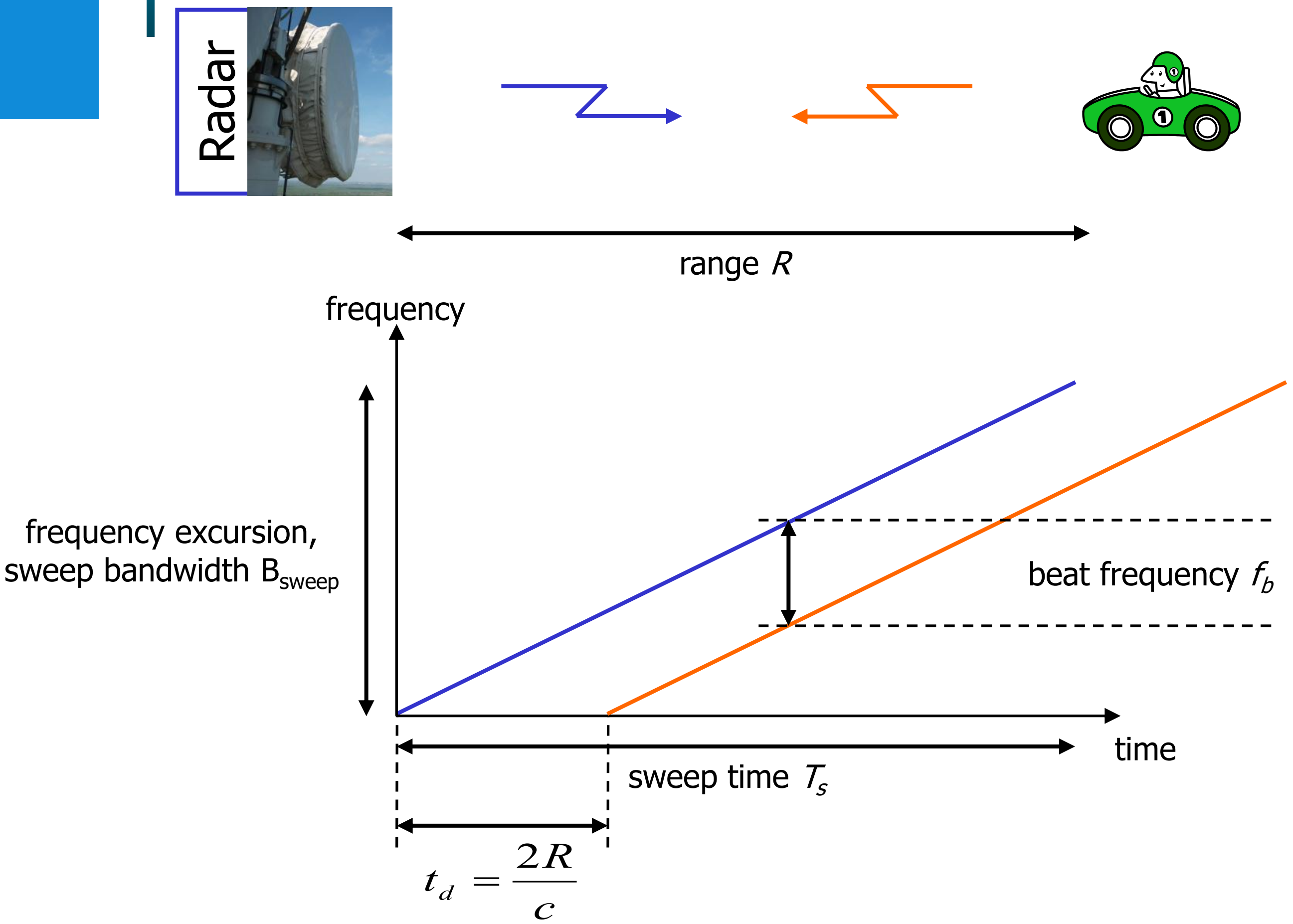




Single target



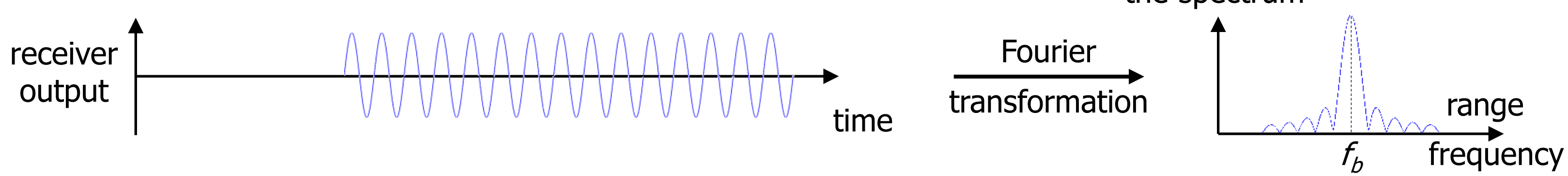
Single target



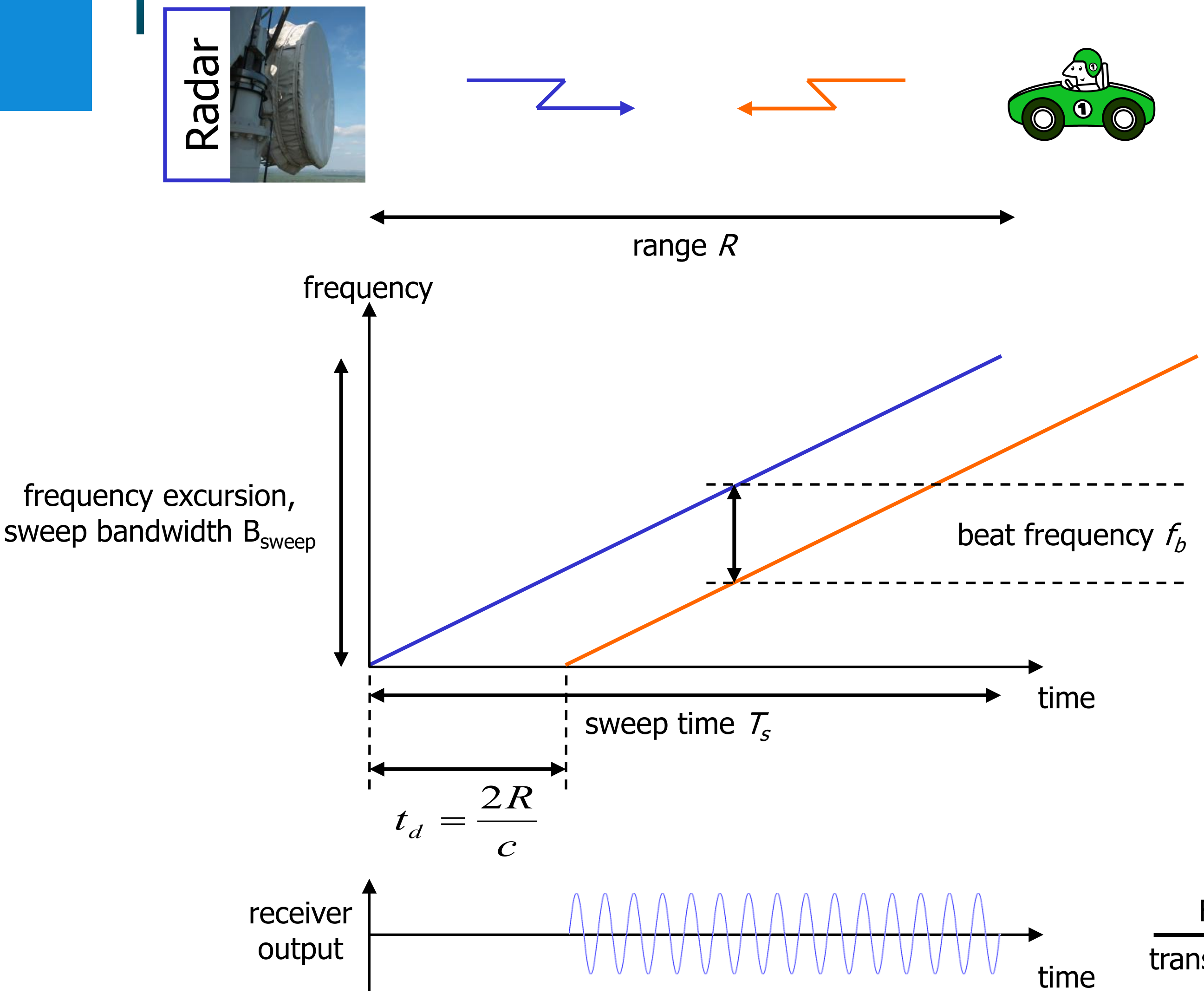
$$\frac{t_d}{T_s} = \frac{f_b}{B_{sweep}}$$

$$R = \frac{cT_s f_b}{2B_{sweep}}$$

$$\Delta R = \frac{cT_s}{2B}\Delta f_b = \frac{c}{2B}$$



Single target



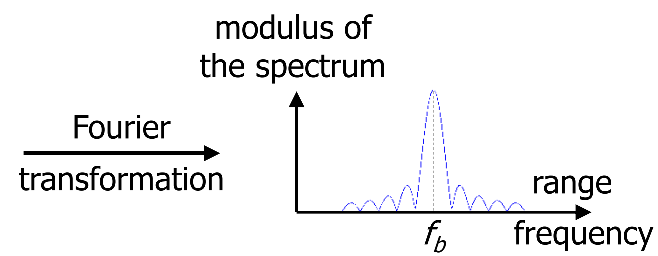
$$\frac{t_d}{T_s} = \frac{f_b}{B_{sweep}}$$

$$R = \frac{cT_s f_b}{2B_{sweep}}$$

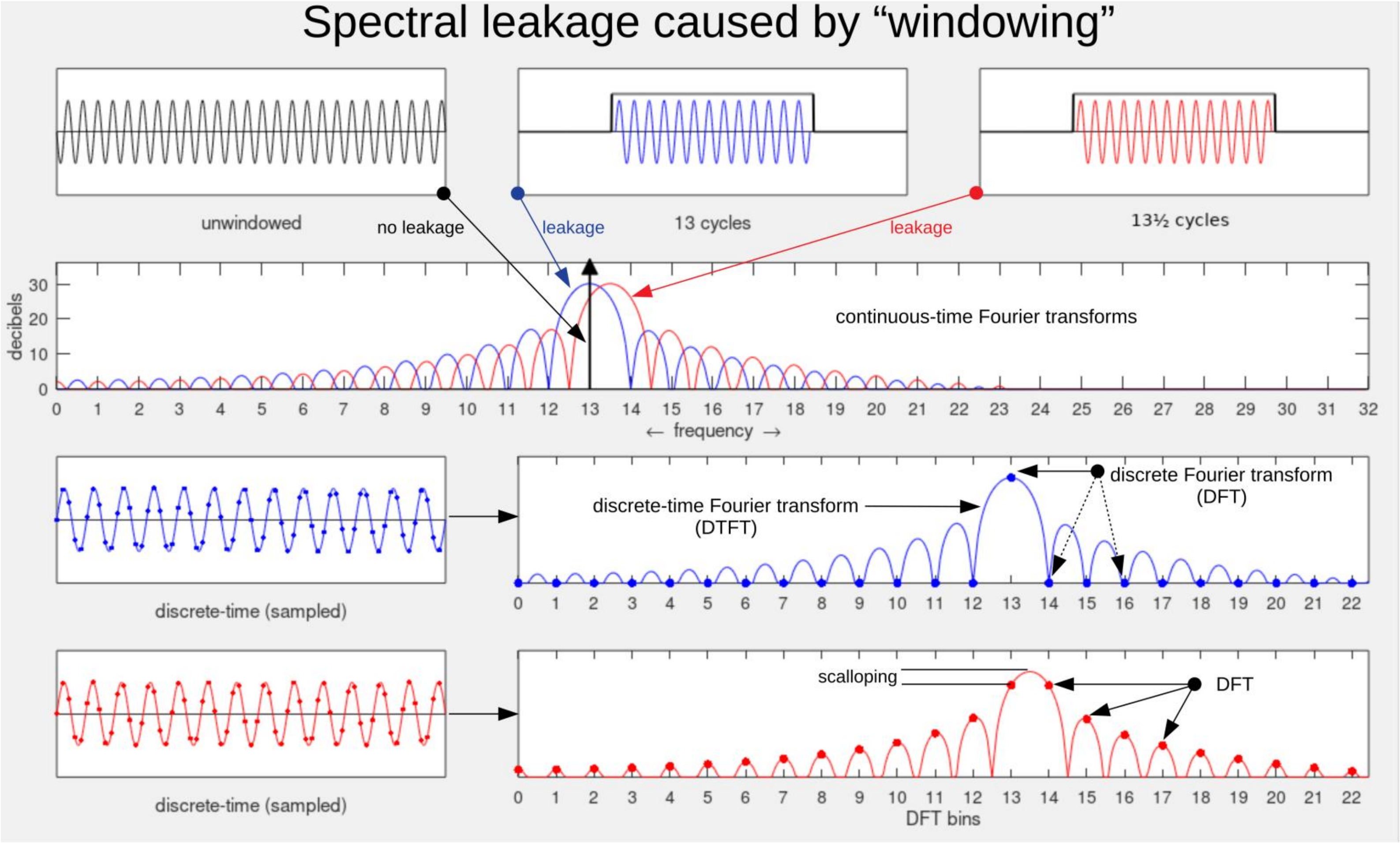
$$R_{max} = \frac{cT_s}{2B} f_{b,max}$$

$$R_{max} = \frac{cT_s}{2B} \frac{f_s}{2}$$

$$R_{max} = \frac{N}{2} \frac{c}{2B}$$



Spectral leakage and windowing



https://en.wikipedia.org/wiki/Window_function#/media/File:Spectral_leakage_caused_by_%22windowing%22.svg

Pulse vs FM-CW

$$P_{noise} = kT_{noise}B_{rec} \qquad B_{rec,fm-cw} = \frac{1}{T_s} \qquad B_{rec,pulse} = \frac{1}{\tau}$$

$$\frac{P_{noise,fm-cw}}{P_{noise,pulse}} = \frac{\tau}{T_s} \cong 10^{-3}$$

Much lower noise floor for FM-CW ….

$$SNR = \frac{C\sigma P_t}{kT_{noise}B_{rec}}$$

$$SNR_{fmcw} = \frac{C\sigma P_t T_s}{kT_{noise}} = \frac{C\sigma P_{mean} T_s}{kT_{noise}}$$

$$SNR_{pulse} = \frac{C\sigma P_t \tau}{kT_{noise}} = \frac{C\sigma P_{mean} T_r}{kT_{noise}}$$

$$P_{mean} = \frac{\tau}{T}P_t$$

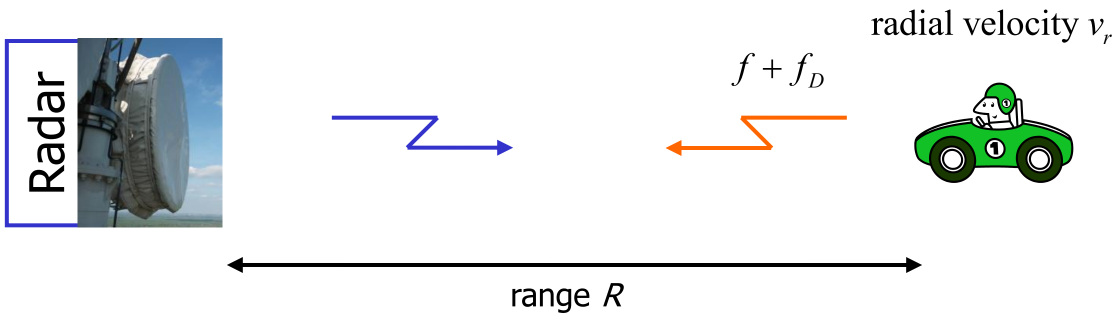
… but similar signal-to-noise ratio

https://en.wikipedia.org/wiki/Window_function#/media/File:Spectral_leakage_caused_by_%22windowing%22.svg

FM-CW and Doppler velocities

Tobias Otto

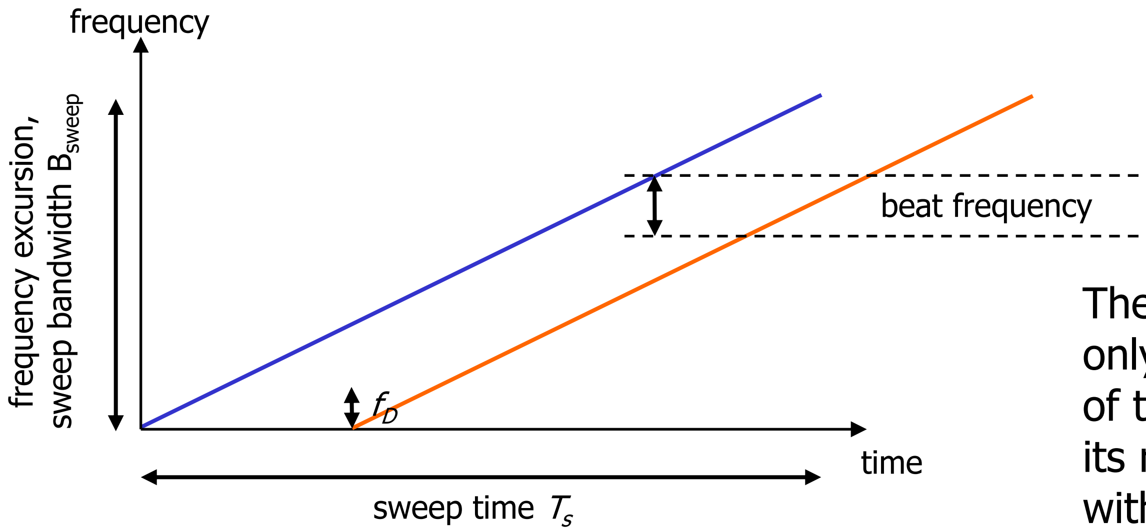
Moving single target



A moving target induces a Doppler frequency shift

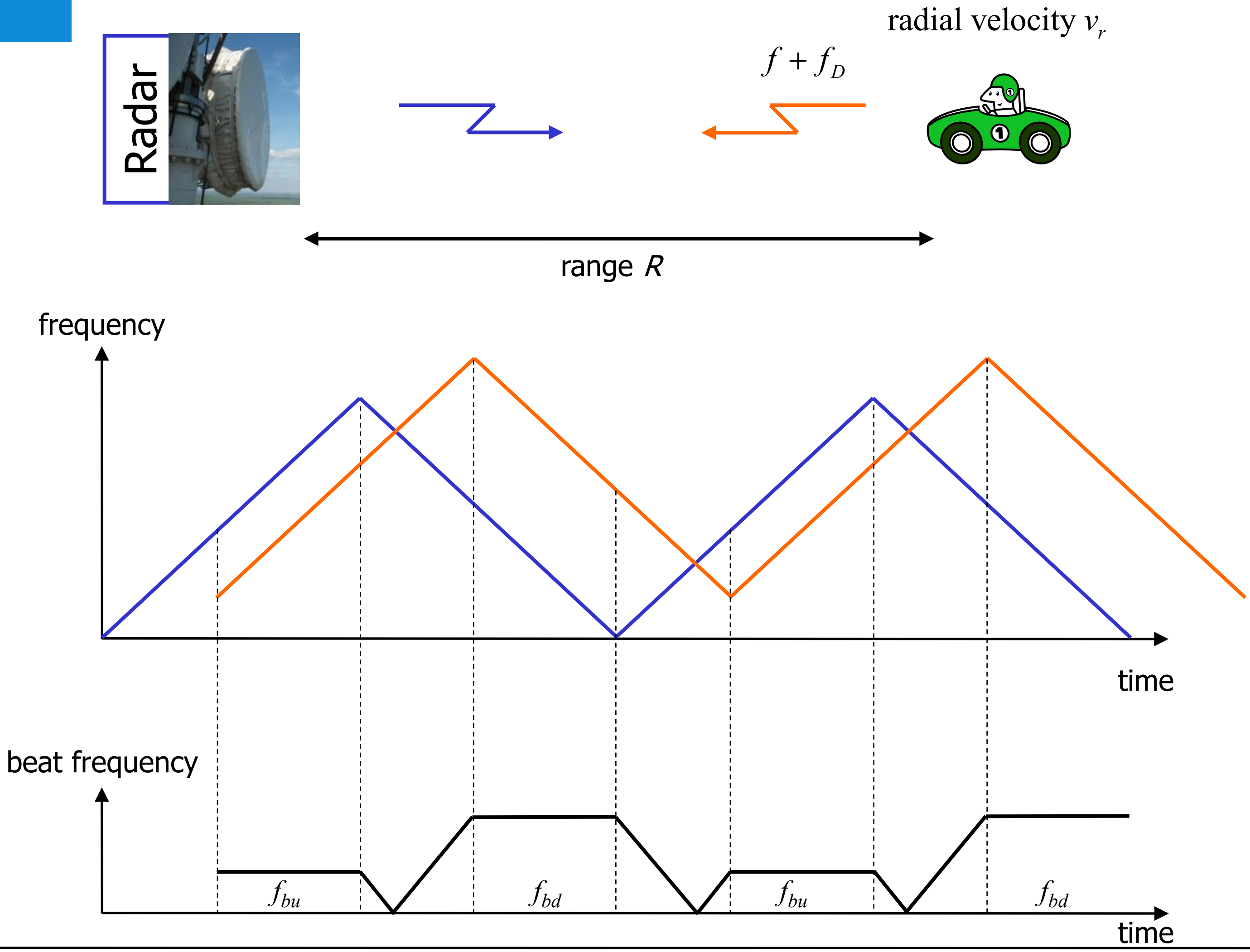
$$f_D = \frac{2v_r}{\lambda}$$

with the radar wavelength λ.



The beat frequency is not only related to the range of the target, but also to its relative radial velocity with respect to the radar.

Moving single target



Beat frequency components due to range and Doppler frequency shift:

$$f_b = \frac{B_{sweep}}{T_s} \cdot \frac{2R}{c}$$

$$f_D = \frac{2v_r}{\lambda}$$

that are superimposed as

$$f_{bu} = f_b - f_d$$

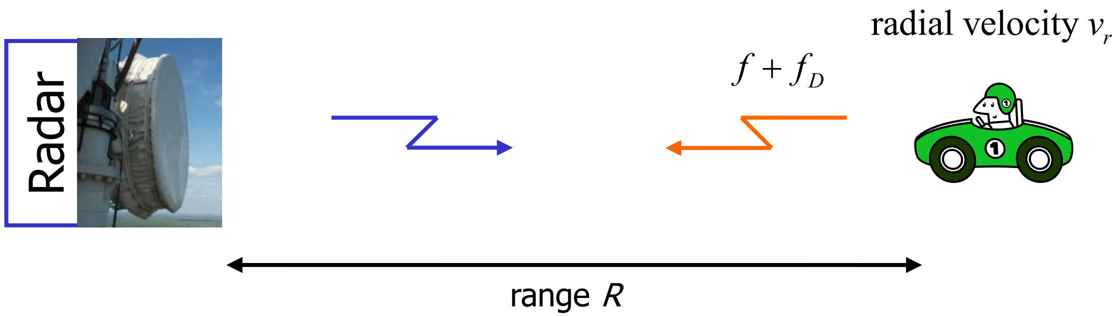
$$f_{bd} = f_b + f_d$$

so range and radial velocity can be obtained as

$$R = \frac{cT_s}{4B_{sweep}}\left(f_{bd} + f_{bu}\right)$$

$$v_r = \frac{\lambda}{4}\left(f_{bd} - f_{bu}\right)$$

Moving single target



$$\frac{f_D}{f_b} = \frac{vcTs}{BR\lambda} = \frac{vfTs}{BR} = \frac{vfTs}{BR} = \frac{94v}{B} 10^{-3}$$

Doppler frequency shift much smaller than beat frequency

$f = 94 \cdot 10^9$

Ts= 10^{-3}

B=B $\cdot 10^6$

R=10^3

Beat frequency components due to range and Doppler frequency shift:

$$f_b = \frac{B_{sweep}}{T_s} \cdot \frac{2R}{c}$$

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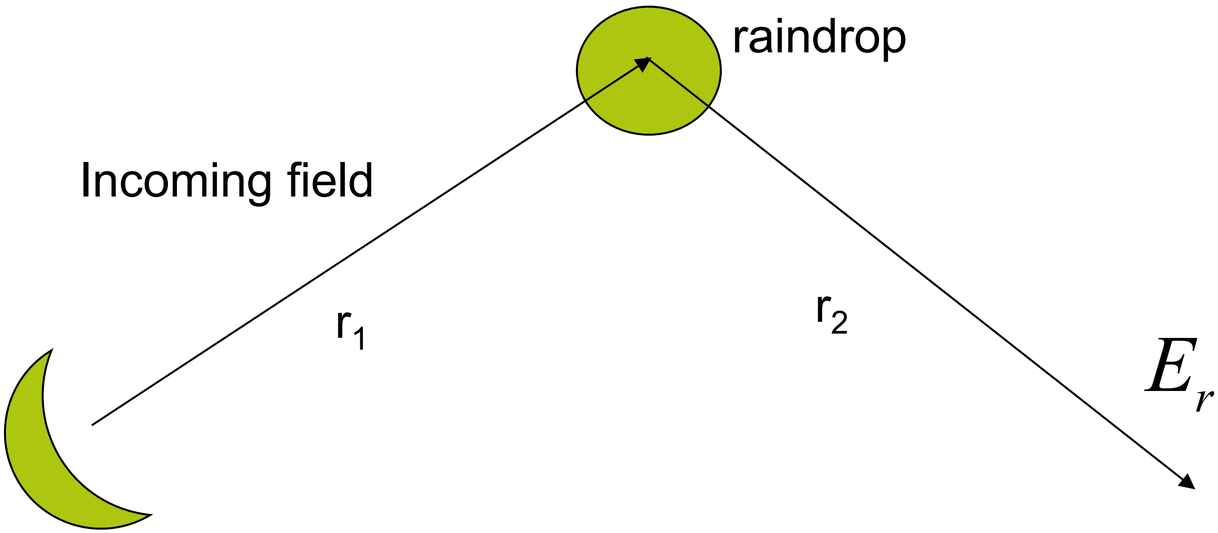
Signal basics weather radar,1



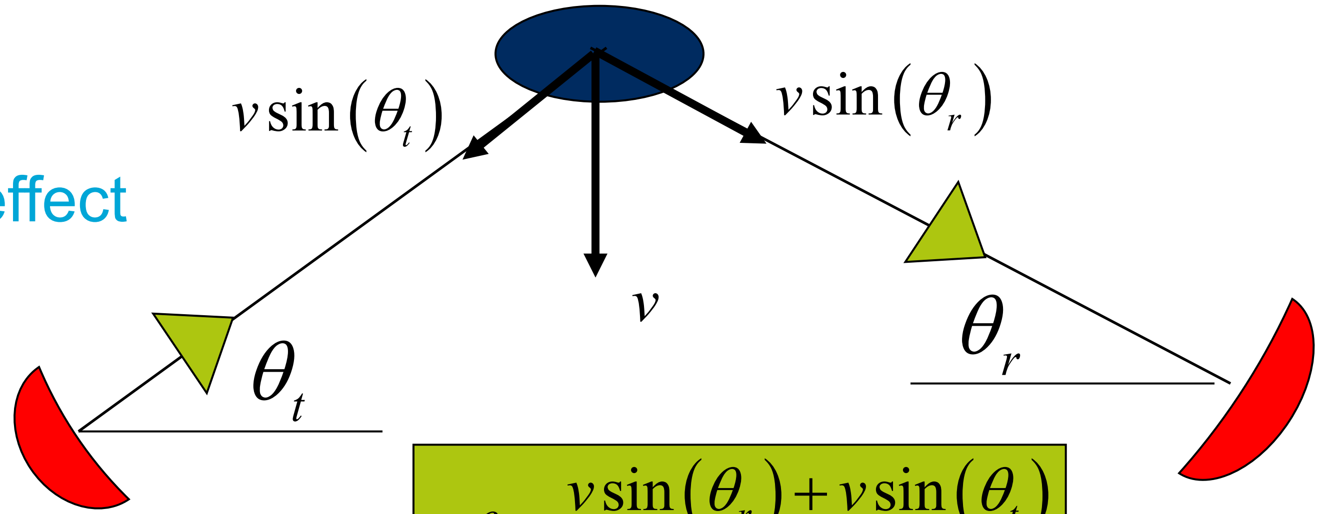
$$E_r \sim E_0 e^{j2\beta(r_1+r_2)}$$

$$\varphi = 2\beta(r_1+r_2) \rightarrow \frac{\partial\varphi}{\partial t} = 2\beta\frac{\partial(r_1+r_2)}{\partial t} = 2\beta(v_1+v_2) = \varpi = 2\pi f$$

$$\beta = \frac{2\pi}{\lambda} \rightarrow f = \frac{2(v_1+v_2)}{\lambda}$$

Signal basics weather radar,2

The Doppler effect



$$\triangle f = \frac{v\sin(\theta_r) + v\sin(\theta_t)}{\lambda}$$

Forward scatter:

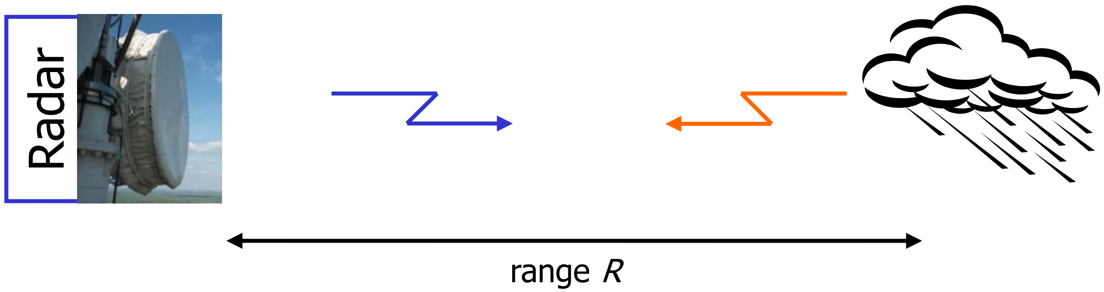
$$\theta_r = \theta_t + \pi \rightarrow \triangle f = 0$$

Backscatter:

$$\theta_r = \pi - \theta_t \rightarrow \triangle f = \frac{2v}{\lambda}\sin(\theta_r)$$

Doppler shift is only representative for the velocity along the antenna beam

Atmospheric FMCW radar



When the expected Doppler frequency shift of the target has a negligible effect on the range extraction from the beat frequency, it can be estimated by comparing the phase of the echoes of successive sweeps.

the phase of the received signal is $\Phi_r = \Phi_t + 2R\dfrac{2\pi}{\lambda}$

the change of the phase of the received signal with time is given by

$$\frac{d\Phi_r}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi}{\lambda}v_r$$

and the change of the phase of the received signal from sweep to sweep is given as

$$\frac{\Delta\Phi_r}{T_s} = \frac{4\pi}{\lambda}v_r \longrightarrow v_r = \frac{\Delta\Phi_r}{T_s}\cdot\frac{\lambda}{4\pi}$$

Signal basics weather radar,3

Maximum phase shift $\varphi_{\max} = \pm\pi$

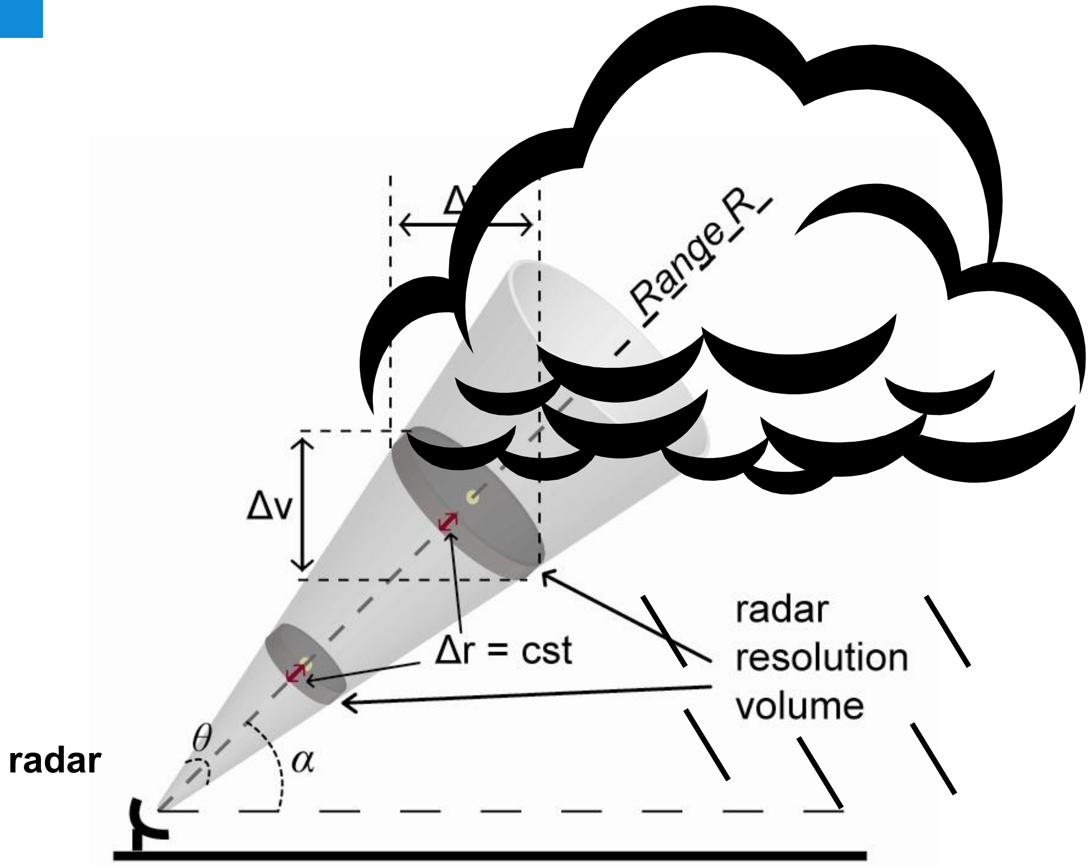
$$\varphi_{\max} = 2\pi f T_0 = 2\pi \frac{2v}{\lambda} T_0$$

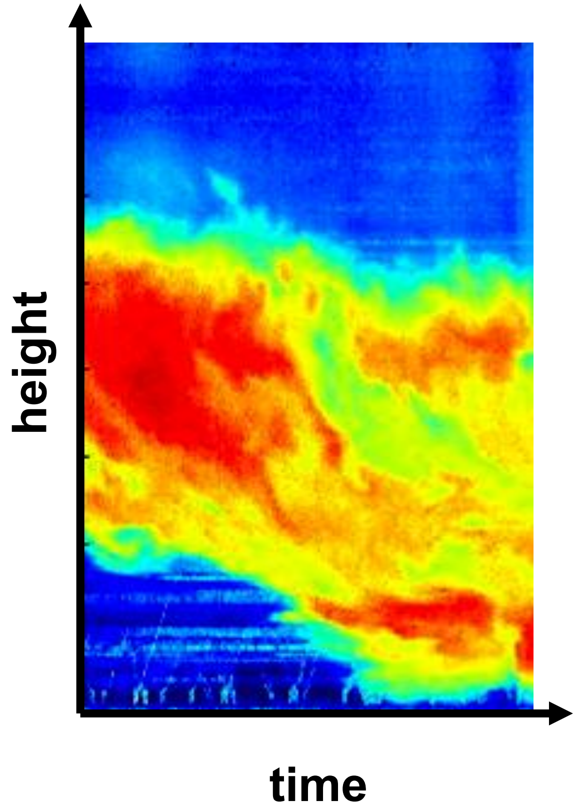
$$v_{\max} = \pm\frac{\lambda}{4T_0}$$

Maximum unambiguous Doppler velocity

Time between two samples

Profiles


Range R
Δv
Δr = cst
radar
resolution
volume
radar
θ
α

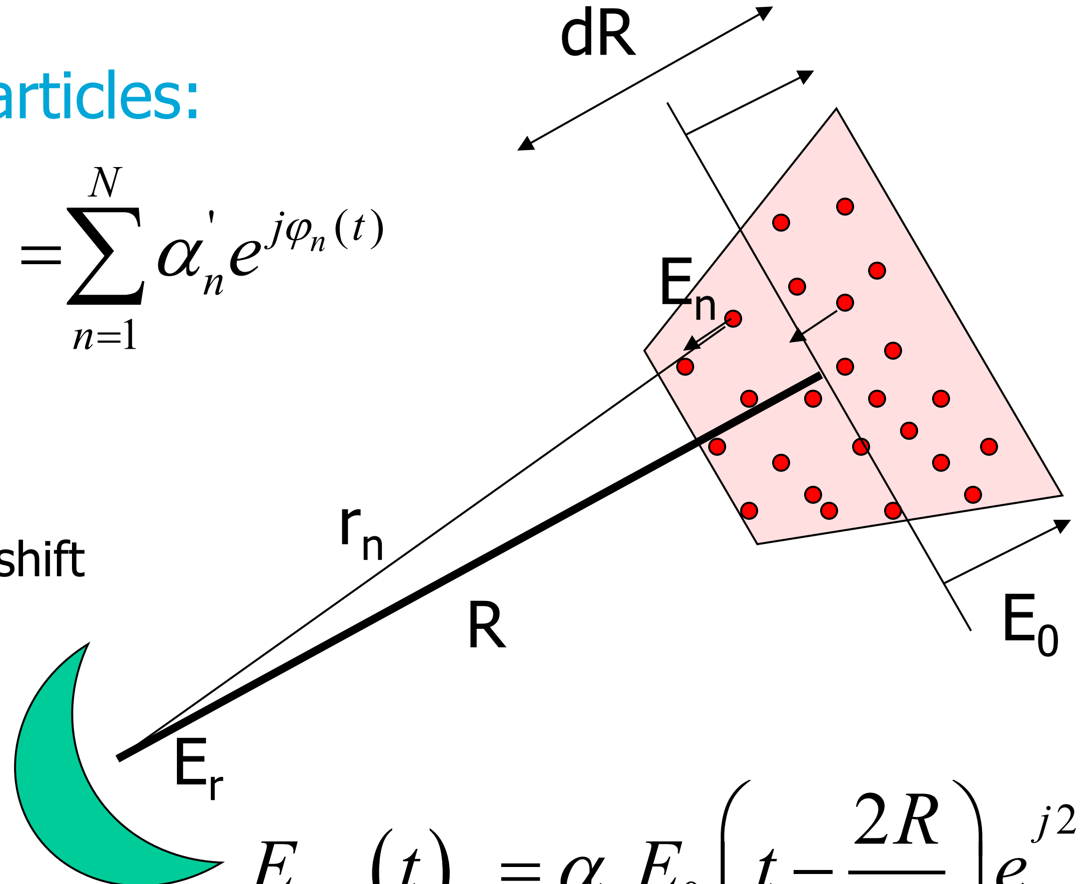

height
time

Scattering by N particles:

$$E_{tot}(t) = \sum_{n=1}^{N} E_n(t) = \sum_{n=1}^{N} \alpha_n' e^{j\varphi_n(t)}$$

$$\varphi_n(t) = \frac{2\pi}{\lambda} 2\left(r_n + v_n t\right)$$

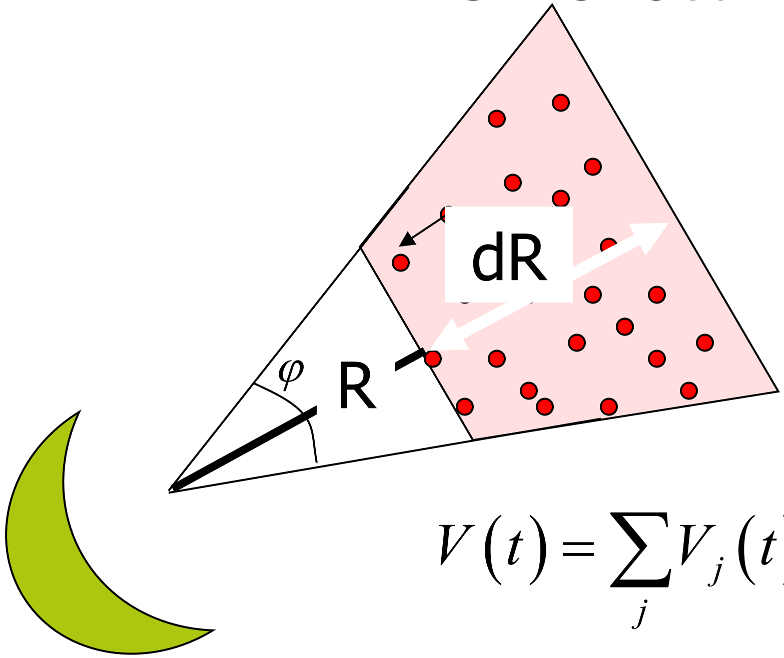
Doppler shift



$$E_{r,n}(t)_r = \alpha_n E_0\left(t - \frac{2R}{c}\right) e^{j2\pi \frac{2v_n}{\lambda} t}$$

$$dR \ll R$$

Signal statistics for volume scattering; one sample, many drops



Every drop j gives a complex voltage Vj
With amplitude and phase

$$V(t) = \sum_j V_j(t) = \sum_j \mathrm{Re}\left(V_j(t)\right) + \sum_j \mathrm{Im}\left(V_j(t)\right)$$

Phase of V(t) uniformly distributed
From statistics: many particles > central limit theorem:
Gaussian distribution of real and imaginary part

Signal basics weather radar

$$E_{tot} = \sum_n^N E_{r,n}(t) = \sum_n^N \alpha_n E_0\left(t - \frac{2R}{c}\right) e^{j2\pi\frac{2v_n}{\lambda}t}$$

Received power

incoherent

$$P = \left|E_{tot}\right|^2 = \sum_n^N \left|\alpha_n\right|^2 +$$

coherent

$$\boxed{2\,\mathrm{Re}\left\{\sum_i^N \sum_{j\neq i}^N \alpha_i \alpha_j E_{0,j}\left(t - \frac{2R}{c}\right) E^*_{0,i}\left(t - \frac{2R}{c}\right) e^{j2\pi \vartriangle f_{ij} t}\right\}}$$

Signal basics weather radar

Doppler information is coded in the signal phase:

a power measurement is not sufficient

complex processing is required to retrieve the particle speed

e.g.Fourier transforms, autocorrelation functions

Basis of spectral processing

one radar cell, one distance

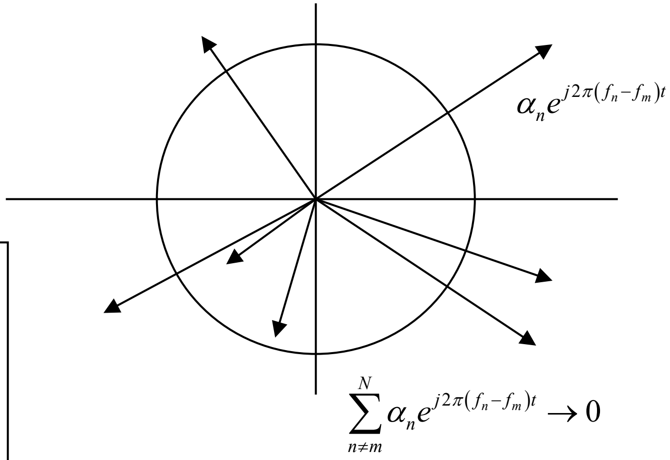
$$E_{tot} = \sum_{n}^{N} \alpha_n(t) e^{j2\pi \frac{2v_n}{\lambda} t} = \sum_{n}^{N} \alpha_n(t) e^{j2\pi f_n t}; \quad E_0\left(t - \frac{2R}{c}\right) = 1 \, for\, sake\, of\; simplicity$$

$$E_{tot}(f_m) = \frac{1}{T} \int_{t}^{t+T} \left(\sum_{n}^{N} \alpha_n(t) e^{j2\pi f_n t} e^{-j2\pi f_m t} \right) dt = \frac{1}{T} \int_{t}^{t+T} \left(\sum_{n}^{N} \alpha_n(t) e^{j2\pi (f_n - f_m) t} \right) dt$$

$$E_{tot}(f_m) = \frac{1}{T} \int_{t}^{t+T} \left(\alpha_m(t) + \sum_{n \neq m}^{N} \alpha_n e^{j2\pi (f_n - f_m) t} \right) dt$$

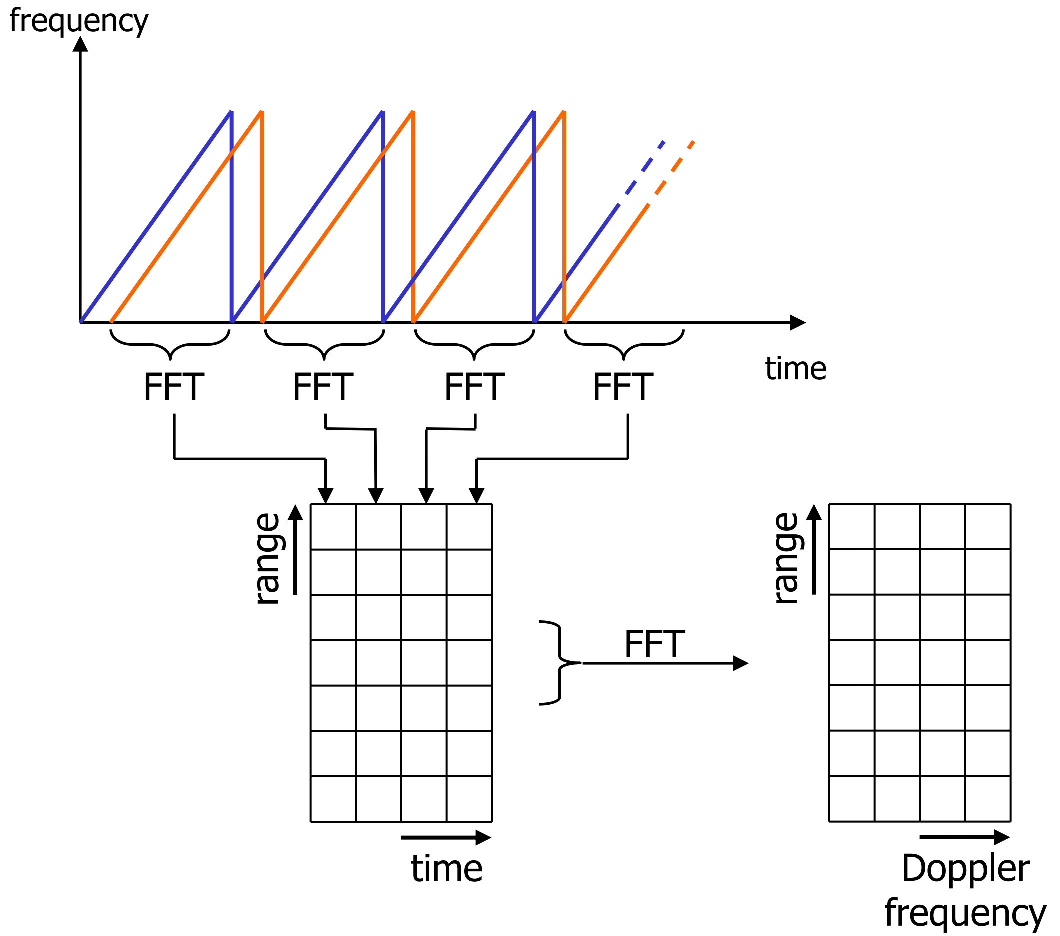
$$E_{tot}(f_m) = \frac{1}{T} \int_{t}^{t+T} \alpha_m(t) \, dt$$

Mean amplitude of all Scatterers with Doppler Frequency *Fm* and velocity *Vm*



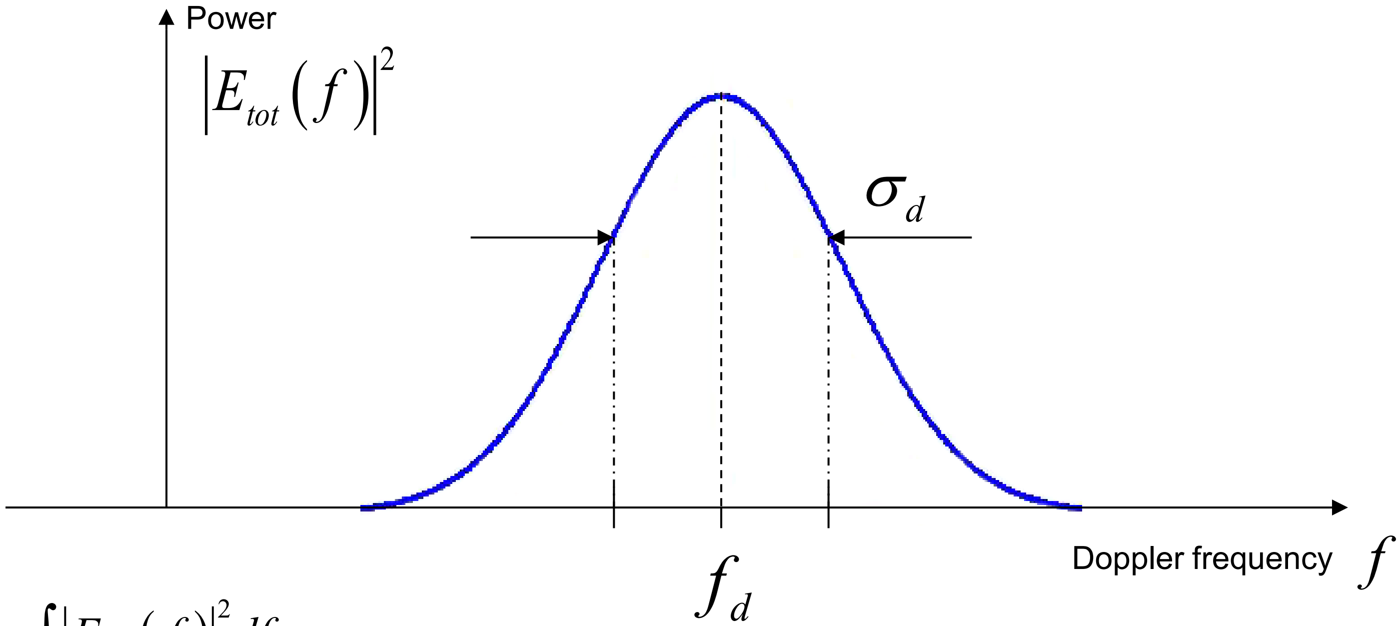
Basis of Fourier transform

FMCW radar signal processing



FFT .. fast Fourier transformation

We obtain a spectrum of all frequencies

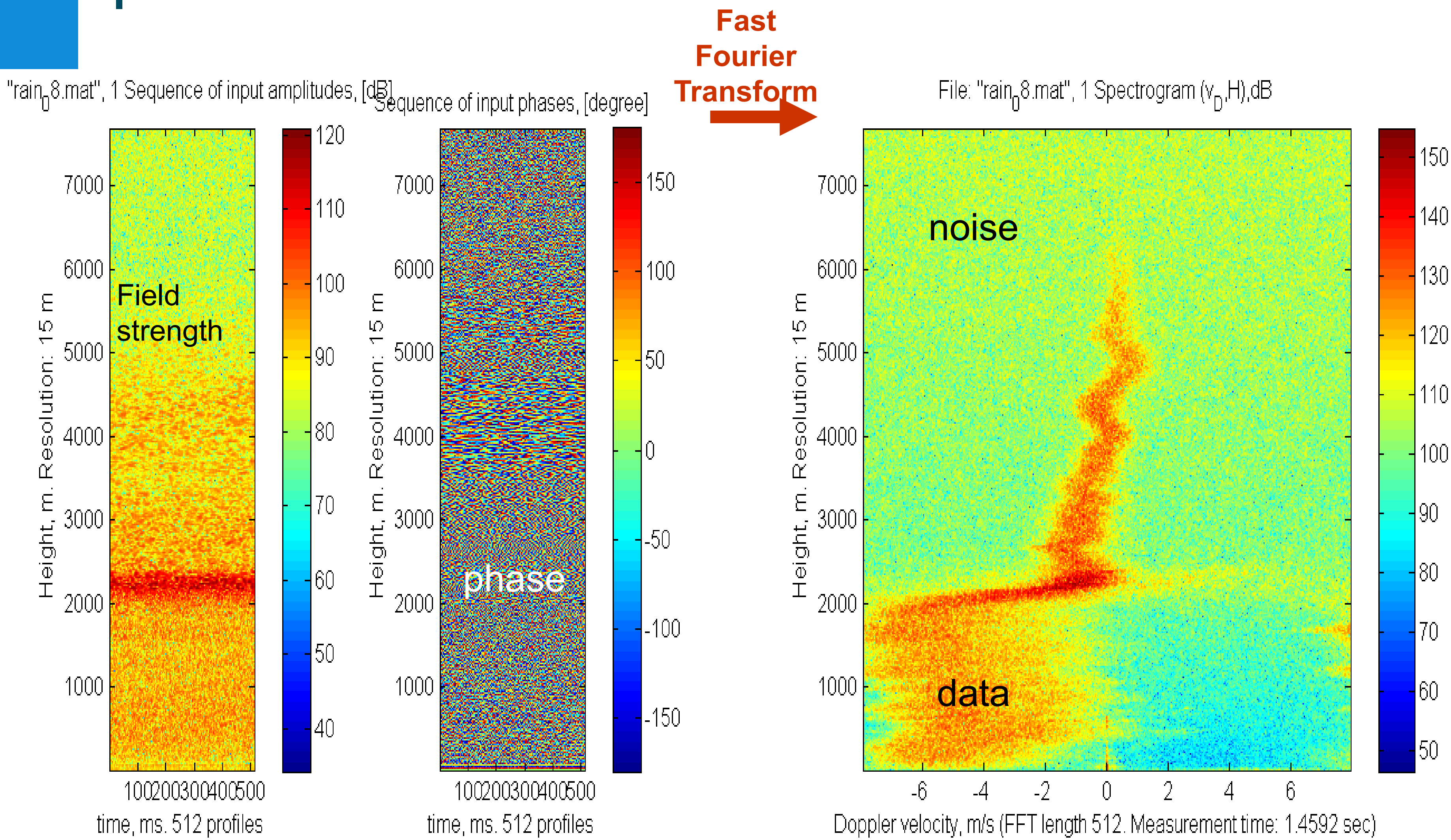


$$P_{tot} = \int |E_{tot}(f)|^2 df$$ ***Total power***

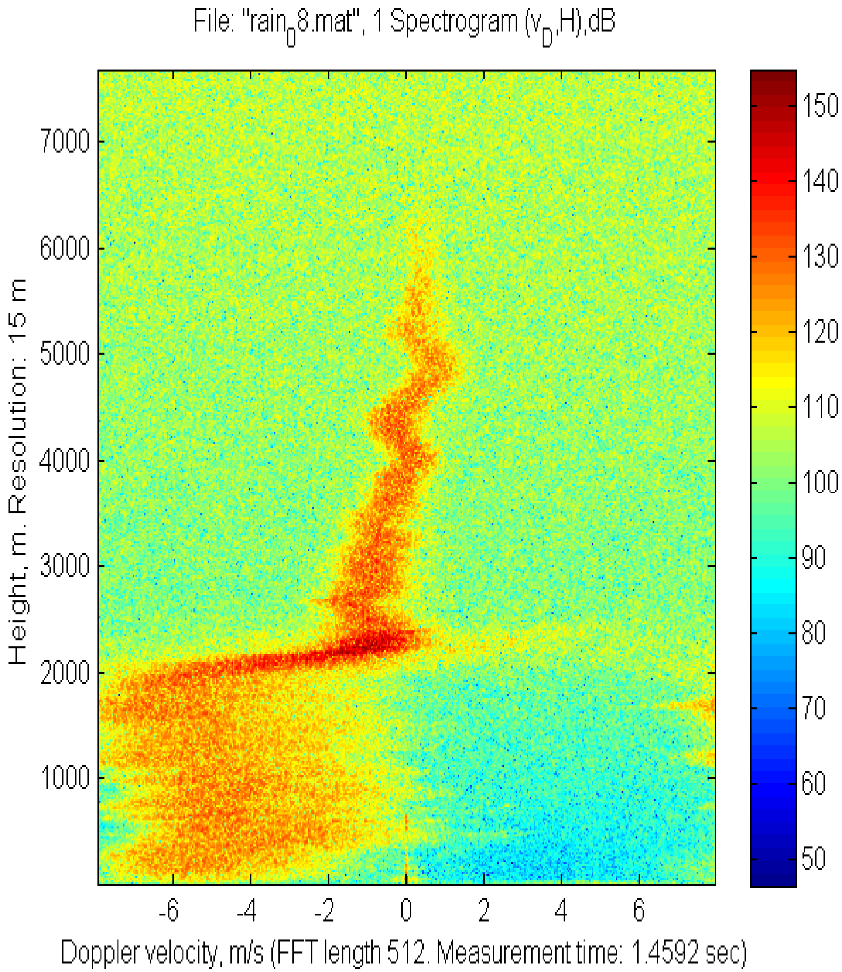
$$f_d = \frac{1}{P_{tot}} \int f |E_{tot}(f)|^2 df$$ ***Mean doppler frequency***

$$\sigma_d = \sqrt{\frac{1}{P_{tot}} \int (f - f_d)^2 |E_{tot}(f)|^2 df}$$ ***Doppler Width***

Raw radar signal and Doppler spectra



Spectrogram = Doppler spectra at every height



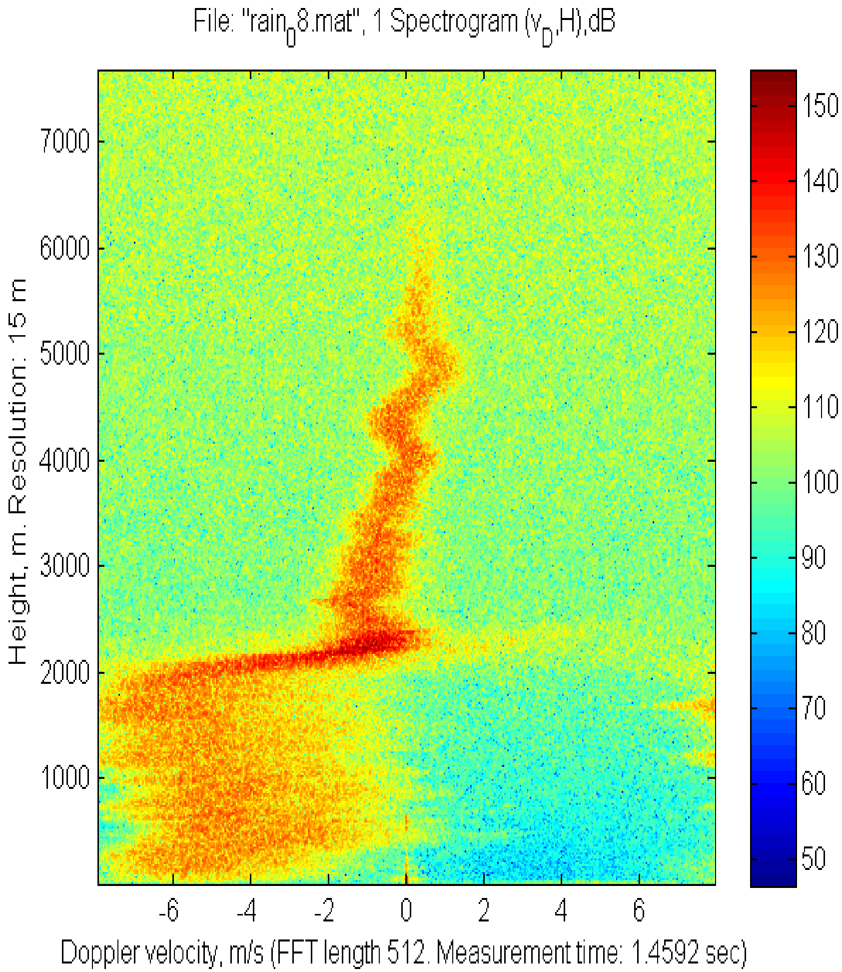
Further necessary steps

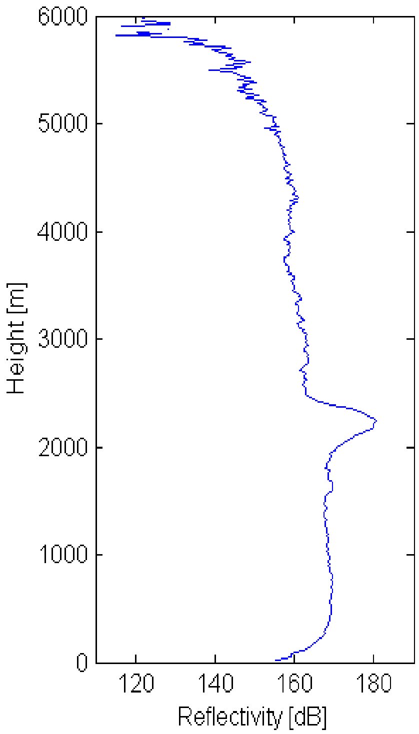
Remove the noise
Signal clipping at a certain threshold

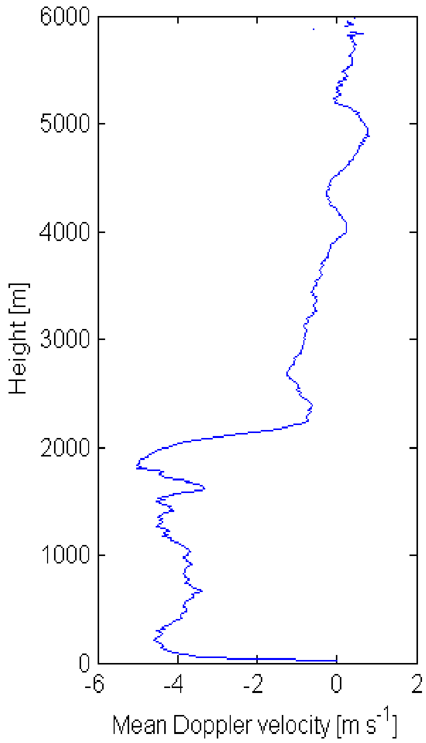
Remove clutter
Filter around v=0 m/s

Calculate the total power, mean doppler Frequency, and the spectral width

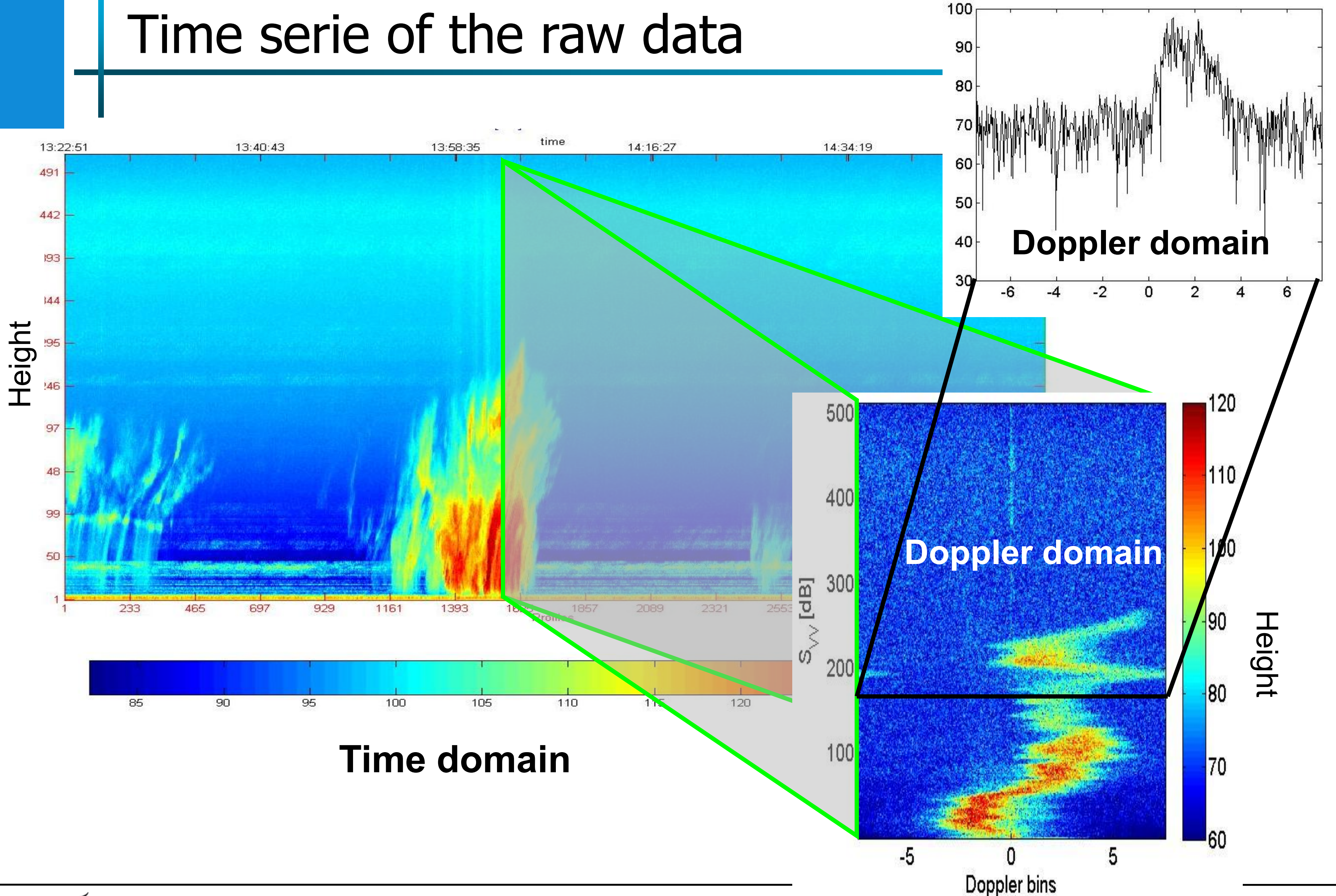
From spectrogram to resulting profiles



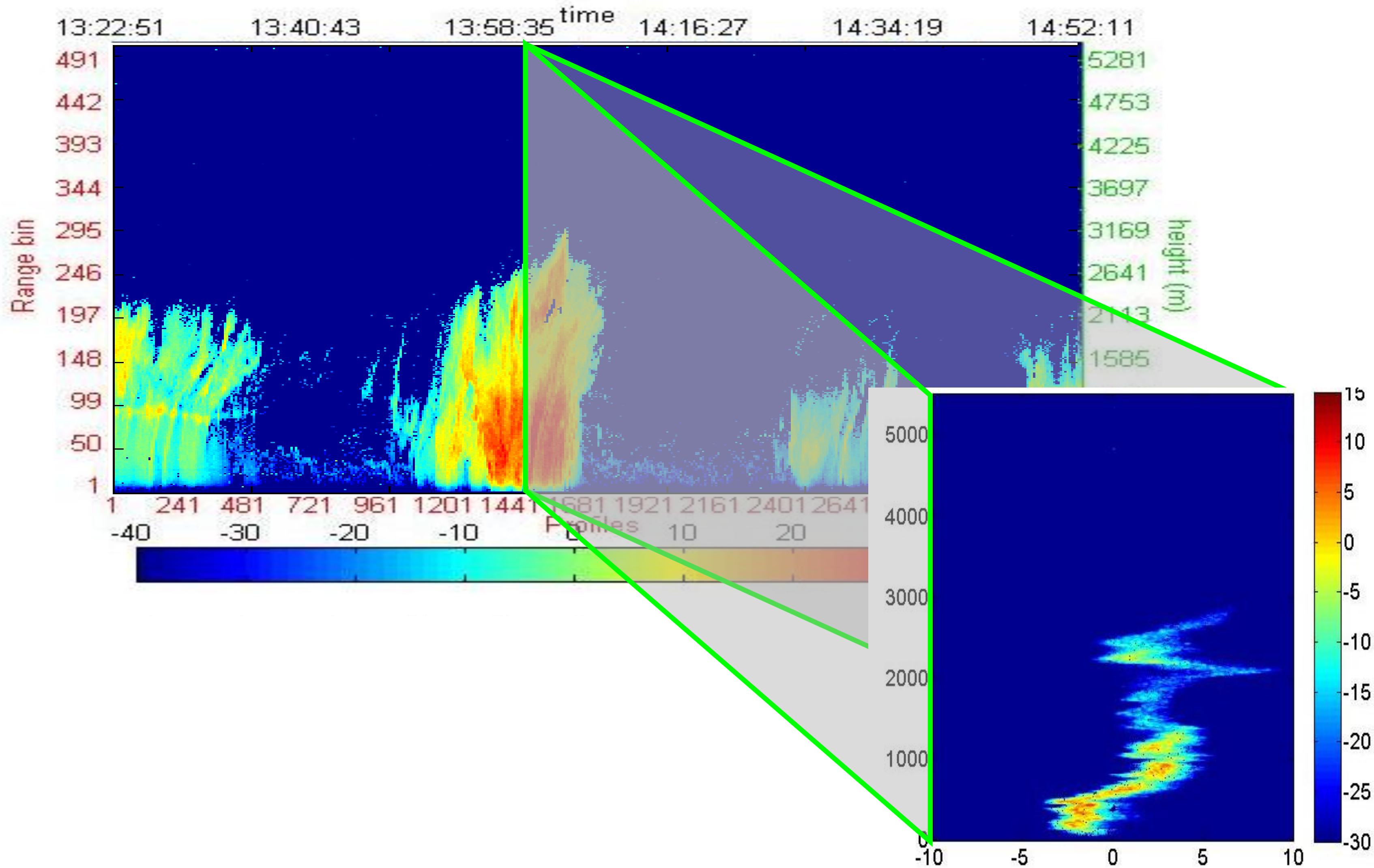


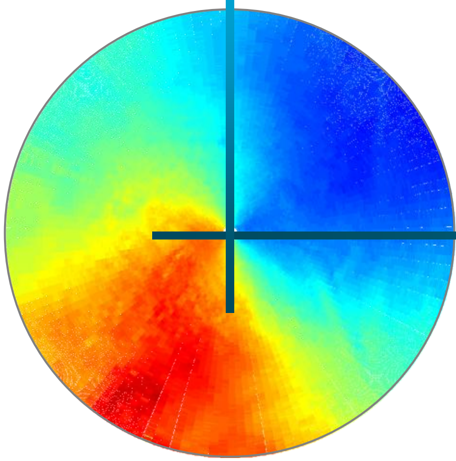


Time serie of the raw data



Processed data





The End

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